# NeMeSiS News 2010

Newsletter of the National Mathematics Summer School

## huan ying! 歡迎

Welcome to the 12th edition of NeMeSiS News! We hope that you enjoy reading it.

If you have any feedback on this issue, or ideas or contributions for the next, I would love to hear from you. You can email me at merryn.horrocks@gmail.com. Thank you to everyone who contributed.

**Merryn Horrocks (editor)**

## A Note From Terry

I've just returned from a visit to Europe. It was really a holiday for me though I did do two semi-mathematical things. In England, I met again some old friends with whom I have worked over many years in the past. One of them is ill now, but it was nice to remember that he and I encountered in some joint research the problem of packing four dimensional space with four dimensional cubes. We thought about building a wall using 4 dimensional blocks. Amazing that one can almost see just what it would 'look' like. Very interesting it is!

In Paris, my wife and I visited the Montparnasse Cemetery seeing among other famous gravesites, those of Simone de Beauvoir and Jean-Paul Sartre, side by side there, as they were in life. There we also saw the grave of Henri Poincaré (1854 – 1912), one of the very greatest French mathematicians. One can look at his work on Wikipedia and elsewhere. He contributed to a myriad of mathematical and scientific fields, including Einstein's Theory of Relativity and the problem of the stability of the solar system (and other n-body gravitational systems), the final version of which anticipated Chaos Theory.

He also is famously remembered for the so-called Poincaré Conjecture, which he made in 1904. This concerns spaces which (approximately) are bounded spaces with no holes and which at each point look like three dimensional Euclidean space. The conjecture was that such a space should be deformable in a precise sense into a three dimensional sphere. If you are interested in seeing exactly what the Poincaré Conjecture is, you should look on the net or in a library. Suffice it to say that this renowned conjecture was verified in 2002 by a Russian mathematician Grigori Perelman. For this he was awarded the most prestigious prize in mathematics, the Fields Medal, in 2006, and then on 1 July 2010, he was awarded a prize of one million dollars, from the Clay Mathematics Institute. This was one of seven Millennium Prize Problems, the solution of any of which will result in a prize of $1,000,000. It is in fact the only problem yet solved. Perelman declined to accept both prizes.

I looked at some of the quotes attributed to Poincaré. Here are three such:

*Guessing before proving! Need I remind you that it is so that all important discoveries have been made?*

This is an old hobby horse of mine. I believe that unless one is prepared to make guesses (even embarrassingly wrong ones at that) one cannot learn and grow mathematically or scientifically. I try to ensure that NeMeSiS students are able and unafraid to guess what might be true, though of course every such guess must follow considerable experimentation and testing in known situations. For example in Number Theory we must make conjectures about what might be true on the basis of calculation (and often boring calculation) with small or not so small numbers.

*A scientist or mathematician worthy of his or her name experiences in their work the same impression as an artist; his or her pleasure is as great and of the same nature.*

*If nature were not beautiful, it would not be worth knowing, and if nature were not worth knowing, life would not be worth living.*

Thanks for all your support with NMSS – it is a wonderful project and one which should continue. If NMSS is to continue, it needs that ongoing support as much as ever.

**Terry Gagen**

## Maths for Thought

Here's a way to multiply two. Let's multiply 8 and 9. First write the numbers and their differences from 10 as follows:

| | |
|---|---|
| 8 | -2 |
| 9 | -1 |

Now there are four different ways to get the first part of the answer:
8 + 9 – 10 = -2 -1 + 10 = 8 – 1 = 9 – 2 = 7.
The second part of the answer is (-2)x(-1) = 2.
So we have our answer: 72.
It also works for bigger numbers.
Suppose we want to calculate 27 x 83.
Then we write the two numbers and their differences from 100:

| | |
|---|---|
| 27 | -73 |
| 83 | -17 |

The first part of the answer is
27 + 83 – 100 = -73 – 17 + 100
= 27 – 17 = 83 – 73 = 10.
The second part is (-73) x (-17) = 1241.
We add the 12 to the 10 to get an answer of 2241.
The question is: How/Why does this work?

(from *Alex's Adventures in Numberland* by Alex Belos, 2010, Bloomsbury)

## An Update from Stephen Farrer

I attended NMSS '98 and '99, and enjoyed it a lot, obviously. In 1999 I started a combined Science/Engineering degree, in pure maths and electrical engineering. After I finished, I worked at Canon Information Systems Research Australia (CISRA) doing image processing. Strangely, image processing is a branch of electrical engineering. We're talking about the concepts behind JPEG and MP3. It was a great job, but then I quit, and in 2007 I decided to do an apprenticeship as a minister (of religion), and now I'm in my second year of an arts degree! (Actually, it's a bachelor of divinity.) My brain is being stretched in all sorts of new ways. I'm thankful to God for all the great opportunities I've had, and the wonderful mathematical world that he has made and that we get to live in!

## NMSS Babies

Congratulations to David Harvey (NMSS 97-98) and Lara Ford (NMSS 95-96) on the arrival of their son Zachery.

At the moment, Lara is engaged in a fellowship in Allergy and Immunology at Mt Sinai Hospital in New York. David is a postdoc at the Courant Institute of Mathematical Sciences at New York University, working in computational number theory and arithmetic geometry. Their son Zachary is nine months old and working on more difficult things like standing up without falling over. All three enjoying living in Manhattan with a population density 13 times that of Sydney. The family will return to Sydney in June 2011, where David will be taking up a Senior Lecturer position in the School of Mathematics at NSW University.

We know of two other NMSS families. Stephen and Varinia Hardy (NMSS 89) also met at NMSS. They have two children, Laura (8) and Elanor (5). Marcus Brazil (NMSS 82/83) and Jacinta Covington (NMSS 77) also have two children, Rueben and Natasha.

Do you know of any other NMSS families? We'd love to hear about them!

# Computational Fluid Dynamics

Fluid Dynamics is the study of the motion of fluids. It is based on a relatively simple set of equations, called the Navier-Stokes equations. These are easily derived by considering conservation of mass and momentum in a fluid - simple concepts a high school student can understand - but it leads to a few non-linear terms in the set of equations which have far reaching consequences.

By far the most diabolical consequence of the non-linear terms is turbulence. Turbulence is a cascade of fluid vorticies with large eddies giving energy to smaller ones and vice versa, all churning and bubbling in continuous motion. The range of size of these turbulent eddies is huge - in weather flows, for example, the largest eddies can be 2000km across, and the smallest eddies far less than a millimetre, with every size in between - and they are all interacting and transferring energy between each other. Most things involving fluid flow involve turbulent flows, from driving your car to exhaling your breath, but a complete understanding still eludes researchers.

An understanding of turbulence remains one of the great unsolved mysteries of mathematics. Werner Heisenberg, Nobel Laureate in Physics for his work on Quantum Mechanics, said, "When I meet God, I will ask him two questions: 'Why relativity?', and 'Why turbulence?'. And I really believe he will have the answer to the first."

Indeed, for the Millenium Prize Problems (seven fundamental unsolved mathematical problems), an understanding of the nature of turbulence was deemed too hard, so a lesser problem regarding the Navier-Stokes equations was posed - simply that for a defined initial condition, there exists a flow field which is smooth and bounded (that is, it decays over time and does not show perpetual motion). While this seems obvious from observation of fluid flows, it remains unproven mathematically. If you can prove it then your prize is $1M!

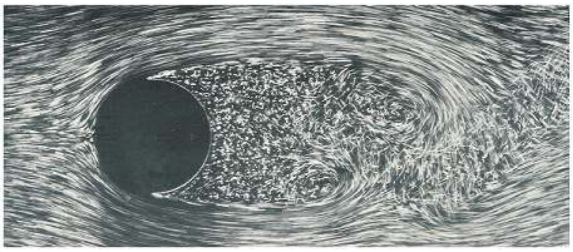
Yet despite our lack of fundamental knowledge of turbulence engineers and scientists are called on to design aircraft, yachts and pipelines; all of which have highly turbulent flows. To simplify the intractable turbulence in these flows turbulence models have been developed, these are empirical curve fits and approximations and are tuned to work reasonably well for a specific type of turbulence in a specific type of flow.

The volume being studied is divided up into small elements (this is called meshing) and a modified form of the Navier Stokes equations applied where the turbulence effects are replaced with an empirical model. The solutions are determined iteratively, often requiring billions of iterations and lots of computing power before a solution is reached. The trick is setting up the mesh and equations so the mathematics converges to a solution which represents reality - otherwise your results are inaccurate at best and rubbish at worst.

A good simulation model can reliably simulate the turbulent flow and optimise the design. Everything from modern aircraft, cars, stadiums to rockets are designed using these computer models. It also produces beautiful images!

**Glenn Horrocks**

(a) visualizing turbulent cylinder wake at Re=10000
Courtesy Thomas Corke and Hassan Nagib; from *An Album of Fluid Motion* by van Dyke (1982)



(b) a closer look at Re=2000 – patterns are identical as in (a)
Courtesy ONERA pic. Werle & Gallon (1972) from *An Album of Fluid Motion* by van Dyke (1982)

# NMSS 2010

2010 marked NMSS' second year at John XXIII College which has well and truly begun to feel like home. Again, the food and the accommodation were excellent, and we had the added bonus of a new plasma TV and a foosball table to boot!

For the 42nd NMSS, in an unprecedented event, we were lucky enough to have both Terry AND Robbie as our number theory lecturers. Terry took the reins for week one and Robbie was at the helm for week two. Needless to say, the number theory lectures were excellent. The only minor hiccup was the disappearance of the Green Martian midway through NMSS, leaving the problem – to whom were we going to explain the mysteries of a*0=0? Despite this, we managed to navigate through the axioms with little drama. The other courses for 2010 were also excellent: Chaos Theory with Leon Polodian, Counting Measurement and Information with Steve Lack, The Road to Catalan with Leanne Rylands, Topology with Ben Burton, Discrete Harmonic Functions with Paul Norbury and Quadratic Numbers with Jim Borger.

It also seemed appropriate that it was at the 42nd National Mathematics Summer School that we discovered the REAL Answer to Life, the Universe and Everything – Euclid's Algorithm (at least, according to Robbie). In addition, 2010 was the year that saw the rise of the NMSS Running Team. This (initially) rather large group braved early morning Canberra to run to the sights of our most beloved nation's capital – Parliament House, Lake Burly Griffin, The Australian Museum, Black Mountain Tower and the Swimming Pool.

As ever, NMSS 2010 went by in the blink of an eye – a blur of Rubik's cubing, schnapping (and for a brief period of time, Pokémon schnapping), soccer, basketball, running, paddle boating, (not) swimming in the creek, Harry Potter rip offs, groundhogs, bears, Captain Planet, fondue, picnics, oranges in Rome, ice skating, stalking, rice and, of course, lots and lots of maths. The 42nd NMSS was a great success both mathematically and otherwise and many thanks must go to all involved for making it such an enjoyable and fulfilling experience for all!

**Gemma Moran-Gibson**

## CAN YOU HELP?

**As you are aware, the NMSS needs all the support that you can give. The school is now without a major sponsor and only remains viable because of the donations of past students and their parents.**

**I urge you to make a tax deductible donation if at all possible. Cheques should be made out to the Australian National University (NMSS) and sent to Professor T M Gagen, Director NMSS, School of Mathematics and Statistics, University of Sydney, NSW 2006.**

**Thank you for your continued support of NMSS.**

**EGs of 2010**