



NEMESIS NEWS

ANU-AAMT NATIONAL MATHEMATICS SUMMER SCHOOL 2020

The newsletter for the 2020 National Mathematics Summer School takes a different approach this year, with a slight focus on mathematics education. From 2001 in NSW it was no longer mandatory to study mathematics in years 11 and 12, and since then the uptake of any mathematics in those year levels has dropped from 91.8% in 2001 to 80.8% in 2009, and even further to 70.7% in 2017.¹

What is deterring today's Australian students from studying mathematics? Is it the nature and content of high school mathematics? Some might wonder: *Is school maths really maths?* Emily Thompson writes an opinion piece on the topic, exploring the nature of high school mathematics. The philosophy of learning at the National Mathematics Summer School focusses on discovery and problem solving; could such an approach in high schools see a resurgence of interest in the subject?

On the other hand, Dr Norman Do considers current popular trends and ideas around the pedagogy of teaching mathematics in *Learning to teach (and teaching to learn)*.

In a world that is moving ever-faster, those no longer learning mathematics at secondary or tertiary level but still passionate about the subject can look to Yudhi Bunjamin's discussion on *Great mathematics podcasts* for some interesting recreational mathematics to listen to while on-the-go.

We also include a brief recount of NMSS 2020 as well as a number of reflections by students, one of which is a poem inspired by the student's experience at NMSS 2020.

Ellena Moskovsky, Editor

¹ Sikora J. Advancing women in Australian mathematics. *AustMS Meeting keynote address*. 2019.

A NEW PERSPECTIVE ON AN OLD TRADITION

NMSS 2020 – NOT A CLASSIC NMSS EXPERIENCE

The National Mathematics Summer School (NMSS) 2020 was a markedly different experience this year. The bushfires that raged across the continent of Australia in the summer of ‘19–‘20 forced it to be so. A week before the scheduled start of the summer school, 74 students were waiting in anticipation to attend what is perhaps considered the most prestigious mathematics camp in Australia. And yet, no fewer than 36 hours before the commencement of the summer school, after a series of increasingly ominous emails about the situation in Canberra, we all received an email informing us that the two-week NMSS we were expecting to attend in Canberra would not be going ahead.

“Did these 70 bright-eyed students get the full NMSS experience?”

It is a testament to the competence and sheer dedication of our amazing director and team of staff that it was possible to hold a one-week NMSS in Melbourne instead. It was the first time in 50 years that NMSS was not a two-week event held in Canberra. Despite the last-minute cancellation and reorganisation, I was astonished to learn that only 4 students did not end up attending the revised summer school – a whopping 70 students confirmed their attendance.



NMSS senior staff, left to right: Emily Thompson, Ben Burton, Michael Smith

Those of you that have attended NMSS previously, and in particular staff members who have been attending NMSS off-and-on since they were students, will wonder whether a one-week version of NMSS could possibly live up to the two-week experience.

So did these 70 bright-eyed students get the full NMSS experience? These students repeatedly heard of events they were missing out on compared to the “usual two-week NMSS”, the biggest of which would be the second week of the number theory lectures, but which also included the second afternoon course, the second alumni lecture, and the Saturday visit to a Canberran landmark of their choosing, to name a few.

But what is NMSS really about? One of the biggest aims of the summer school is to give students a glimpse of what mathematics research is like, and in this regard the one-week NMSS does this job certainly much more than half as well as the two-week NMSS, if not just as well. Within one week the students were shown what it is like to question and discover, to do research in mathematics themselves, and to think very hard about apparently trivial problems.

The other, in my view, fundamental part of the National Mathematics Summer School is the cultural aspect. NMSS has always been a welcoming place for everyone; a place where students who may be cast out as ‘nerds’ in their own school feel like part of a society, a place where many walk away with lifelong friends. In this aspect, I cannot compliment the returning ‘Experienced Group’ highly enough.

did a tremendous job of creating and establishing this welcoming and friendly culture early on. Their effort allowed us to skip the nervous awkwardness that students tend to exhibit in the first week, and jump straight into the camaraderie and warmth usually reserved for week two.

“ [The Experienced Group’s] effort allowed us to ... jump straight into the camaraderie and warmth usually reserved for week two.

So, did these students get the full NMSS experience? I think so, yes. They learnt some interesting mathematics, they thought deeply about simple things, and they learned what it was like to share a passion with others their age. They made lifelong friends.

Ellena Moskovsky, NMSS Tutor



Three Experienced Group students and their tutor from 2019, left to right: Charlotte, Reef, Emily, Joanna

Each year, around 12 students who attended NMSS the previous year return as the Experienced Group. This year that group



DIVIDING BROWNIES

A father bakes brownies in a rectangular pan as an after-school snack for his two daughters. Before his daughters get home, his wife comes along and removes a rectangle from somewhere in the middle, with the sides of the rectangle not necessarily parallel to the sides of the pan. How can he make one straight cut and divide the remainder of the brownies evenly between his two daughters so that they both get the same area?



This puzzle was sourced from Francis Su’s book, Mathematics for Human Flourishing.

GREAT MATHEMATICS PODCASTS

A PhD student in mathematics and statistics at UNSW, Yudhi Bunjamin was a first-time NMSS tutor in 2020. Yudhi is also part of a team that is involved in designing mathematics outreach for primary and secondary school students.

No matter what subjects we are interested in, we all like to engage with our craft. If you like English, then you probably read a lot of novels in your spare time. If you like physics, then you might spend a lot of time looking at the night sky through a telescope. If you like history, then you might travel a lot or be a frequent visitor of museums. But it's usually not so obvious what people who like mathematics might do in their spare time to engage more with mathematics that doesn't involve doing more problems.

One great way to engage more with mathematics is to listen to some podcasts about it. You won't learn about the proofs to new theorems this way but there are many great podcasts out there which will discuss the deeply philosophical question of what mathematics is really about as well as keep you up-to-date with the latest advances in mathematical research and teaching.

What follows are some of the mathematics-related podcasts I regularly listen to which I highly recommend. This might be a great way to keep engaging with mathematics after NMSS, especially if you don't go on to pursue mathematics later on in your career. For each of them, I highlight an episode which I particularly liked.

Note that there are lots of other podcasts out there which are a bit more focused on a particular topic. For example, Robert Nguyen who is currently doing his PhD on statistical methods in Australian Rules Football has a

great podcast on the statistics of sports analytics called *Chilling with Charlie* (player.whooshkaa.com/shows/chilling-with-charlie).

If you speak some other languages, there are also lots of good podcasts out there in different languages. Kevin Limanta, a fellow Indonesian who is also currently doing his PhD at UNSW, has a podcast called *Matematika, bisa apa sih* (listennotes.com/podcasts/matematika-bisa-apa-sih-kevin-limanta-xbv2H6-YGp) where he interviews Indonesian mathematics PhD students around the world.

Sum Of All Parts

abc.net.au/radionational/programs/sum-of-all-parts

Hosted by Joel Werner who is the Audio Lead for ABC Science, this podcast is about “the amazing and powerful numbers all around us, how they work, and the people who work with them” and covers an insanely broad range of topics. The podcast is very well-produced and is very much non-technical, making it great and easy listening for when you're on the go. However, I think the best thing about this podcast is that when you listen to it, it doesn't feel like a podcast about mathematics. I find this to be a constant reminder that mathematics is about ideas and ways of thinking and not just about numbers, even though the podcast is described to be telling “extraordinary stories from the world of numbers”.

One episode to check out: 15.0 An unusual pattern

This episode tells the story of Benjamin Geen, a former nurse who was accused of causing grievous bodily harm to his patients after the emergency department in his hospital observed what they considered to be an unusual number

of patients who suffered respiratory arrests or depressions while he was on shift. It discusses his trial and how some statisticians argue that he did not get a fair trial, partly because what they considered to be “unusual” may not be that unusual after all.

The Random Sample

acems.org.au/podcast

This is a podcast by the Australian Research Council (ARC) Centre of Excellence for Mathematical and Statistical Frontiers (ACEMS) where they speak to some mathematicians and statisticians, mostly in Australia. The episodes cover a broad range of topics including research, mathematics engagement, and equity in mathematics. In many of the episodes they take some time to talk about the journeys that led them to be interested in mathematics and statistics. One thing which I like about this podcast is that it keeps me updated with some of the work being done by mathematicians and statisticians in Australia. Lately, there have been some episodes about how some Australian mathematicians and statisticians have been involved in COVID-19 research.

One episode to check out: MARVEL-lous Casting (Episode 16)

This is an interview with Professor Matthew Roughan about work he’s done with two of his colleagues at the University of Adelaide (one of whom is a friend of mine) on how the cast size in a movie from the Marvel Cinematic Universe could predict the movie’s success.

The Numberphile Podcast

numberphile.com/podcast

If you’re a fan of the YouTube channel Numberphile, then you might want to check out The Numberphile Podcast also hosted by Brady Haran. Unlike the YouTube videos, the podcast focuses more on the lives and careers of the mathematicians being interviewed rather than the mathematics itself. Brady is very good

at asking the kind of questions which someone with no mathematics background would ask, which is why I think this is a particularly good podcast for high school students looking to know more about what a career in mathematics is really like.

One episode to check out: The Badly Behaved Prime - with James Maynard

James Maynard, most well known for his work on gaps between prime numbers, talks to Brady about his early years which led to his current position at Oxford and what it’s like to be working on some of the biggest problems in mathematics.

Stats + Stories

statsandstories.net

Described as “a podcast about the statistics behind the stories and the stories behind the statistics”, this podcast is a great way to learn more about statistics which is an area of mathematics we don’t usually see much of at NMSS. The podcast focuses on the challenges of communicating statistics which are usually the results of some very sophisticated statistical tools in a way that is understandable to the general population but without compromising the accuracy. Interestingly, this podcast is being produced by the Department of Statistics and the Department of Media, Journalism and Film at Miami University. The guests who have been featured on the podcast include some of the biggest names in the statistics world including some of Australia’s most respected statisticians.

One episode to check out: Hey, I’m Working On A Story And Can You Help Me With The Statistics? (Episode 52)

This episode features Professor Rebecca Goldin from George Mason University who is the Director of STATS at Sense About Science USA. She discusses what it’s like to work with journalists who usually do not have much training in statistics to report statistics accurately but also in a way that can be easily understood by readers.

Yudhi Bunjamin, NMSS Tutor

LEARNING TO TEACH (AND TEACHING TO LEARN)

A senior lecturer in mathematics at Monash University, Dr Norman Do was a NMSS student in 1997 and 1998. He has been a NMSS staff member since 2001, including taking the first week of number theory lectures on five occasions.

Well, what a year it's been so far! As an academic mathematician whose lectures underwent a forced migration online and as a parent who has necessarily become more involved in my children's day-to-day learning, I have found myself thinking deeply about the not-so-simple thing we call teaching. Although I was asked to share here some of my thoughts on maths education, I must admit that I am certainly no authority on the matter, but just a humble practitioner striving to do better. So I will merely pass on the wisdom of others in the hope that someone apart from myself might find it interesting.

The educational literature, blogosphere and Twitterverse are awash with well-intentioned dictums, of wildly varying degrees of utility. However, on the odd occasion, one comes across an aphorism beyond whose pithy facade lies a kernel of truth that can help to catalyse constructive change. The following are three examples that have particularly resonated with me during this disrupted year of teaching and learning.

I don't just teach maths, I teach people

All too often, mathematics instruction is delivered in a manner that is cold, heartless, austere and mechanical. Worse still, it can be delivered without regard for the people on the receiving end. I know that I have certainly been in the audience while a lecturer reads out

dense abstract mathematics from slides while I scramble to take notes. Such experiences made me wonder: could I have learnt this without the lecturer? Would the lecturer have taught this without the students?!

I firmly believe that good teaching should appeal to our natural human desires: to make meaning of the world around us, to impose order on it, to stir the curiosity within ourselves, to become explorers of ideas, and simply to play.

Furthermore, a student-centred approach to teaching maths needs to take into account just how much each individual brings along to their learning. This includes their previous experiences with the subject in and out of the classroom, their intuition, their goals and ambitions, and their personal circumstances. It might seem obvious to say that not everyone has shared my mathematical upbringing and experiences, that not everyone seeks to become a professional mathematician like myself. That diversity is indeed a good thing and my teaching needs to reflect that.

Motivation doesn't lead to achievement, achievement leads to motivation

Few people would argue with the fact that motivation and achievement are intrinsically linked, and perhaps more so in the study of mathematics than in most other disciplines. However, there is a strong tendency to err on the side of believing that it is motivation that leads to achievement, when there is mounting evidence to suggest that the direction of causation is stronger in the opposite direction than we think. Thus, to create the virtuous

cycle in which motivation begets achievement and vice versa, we should focus on the latter, rather than the former.

To a certain extent, it is good practice to motivate students to learn, whether by proverbial carrot or proverbial stick. However, this should take second place to teaching students well, scaffolding their learning, giving them opportunities to achieve, and celebrating those initial learning victories. Intrinsic motivation then follows.

Maths educators often dread when a student asks the question: why is this useful? This of course deserves a response, but I claim that there is no need to pre-empt the question and no need to preface each lesson with talk of applications. That discussion should enter naturally and, at least from my own experience, doesn't necessarily achieve the desired goal of motivating students to learn. Most commonly, the student asking why this is useful is one who hasn't fully grasped what is being taught, and that is the issue that really needs to be addressed.

On this note (excuse the pun!), if all you have is the middle C key on a piano, then there's very little of value that you can do. However, that key in the context of the whole keyboard allows you to replicate musical masterpieces and to compose hitherto unheard melodies. So a short and honest answer to why solving the equation $2x^2 - 3x + 1 = 0$ is useful is that, in isolation and without context, it probably isn't!

Don't cover the material, uncover the material

Mathematics education, particularly at the secondary and tertiary levels, is overly concerned with the delivery of content and the assessment of skills. The metronomic pace of a curriculum can be constraining, without allowing students the latitude to flex their creative muscles, to develop the resilience required to flourish in productive struggle. It is

also unforgiving to students who don't keep up, when it is slow and deep thinking that we should value and encourage.

From my experience, the exemplar of a teaching and learning environment that encompasses this philosophy is the National Mathematics Summer School. As with some of you reading this, I had the opportunity to sit through Terry Gagen's amazing number theory lectures, as a NMSS student in a previous millennium. At the time, I knew that what we were learning and how we were learning were markedly different from any previous experience. Yet it took several more iterations of seeing those lectures from the perspective of a tutor to truly appreciate what that difference was.

You see, Terry didn't explicitly tell me what Euclid's algorithm is and he never explicitly proved to me why it worked. He simply showed it to me and that in itself was an invitation to try it out for myself, to wonder why it worked in his example as well as mine, to go down the rabbit hole, so to speak. Then Terry showed me Euclid's algorithm not just for two whole numbers, but for two polynomials, and my mind was blown.

There was no imperative to cover a certain amount of material in the limited timeframe of that wondrous fortnight. Rather, Terry's lectures enabled us and invited us to become mathematical explorers.

Students come to the NMSS at differing stages of their mathematical progression, and they also leave the same way, but hopefully enriched by the experience. They don't all walk away remembering which prime numbers can be expressed as the sum of two perfect squares and they don't need to. (With the help of the internet, you could find out in less than a minute!) NMSS students have instead been given the opportunity to discover mathematics for themselves, to take ownership of those discoveries, and to experience what mathematics and teaching and learning can really be.

Dr Norman Do, NMSS Lecturer

STUDENT REFLECTIONS

WHEN I TOLD MY MATHS TEACHER THAT I WAS APPLYING FOR A TWO-WEEK MATHS CAMP, HE LAUGHED AND CALLED ME A NERD.

But that didn't stop me applying, nor did it diminish the excitement that came with my offered place at the camp. I was keen for the opportunity to experience more than just everyday school mathematics and NMSS did not disappoint.

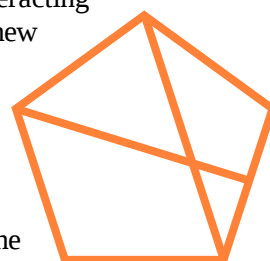
Skipping forward about a month after I accepted a position, the bushfires had arrived in the ACT, and it was decided for the safety of all involved in the camp that NMSS 2020 would be cancelled. Needless to say, I was disappointed but the prospects of the offered week in Melbourne kept my hopes alive. Thanks to the relentless work of our dedicated director, the flights for every student were booked within the week, and we were off.

“I was brought into a family of people who strived to make themselves better, and who incorporated me without a blink.”

What followed was easily the best week of my life. I knew no one else going, and as you can guess, I was quite nervous. But I needn't have feared.

Upon my arrival I was greeted by the friendly tutors who tirelessly urged us to think deeply of simple things, the experienced students who went out of their way to welcome us all and ease our nerves, our welfare officers who proceeded to keep us safe throughout the week, our lecturers who never ceased to broaden our perspective of mathematics and our directors who brought the whole event together.

I was brought into a family of people who strived to make themselves better and who incorporated me into their group without a blink. I learnt not only new concepts but new ways of thinking, of interacting with people and saw a new side to a subject that I thought I knew inside and out. NMSS is an experience that I will never forget and I am extremely grateful for the opportunity I was offered.



Alice Hughes, NMSS 2020 Student



NMSS 2020 staff, left to right: Andy, Isabel, Ell, Olivia, Michael, Ben, Emily, Tom, Norm, Brendan, Yudhi, Sean, Merryn, Ellena, Jonathan, Leanne (absent)

PEOPLE ALWAYS SAY THAT HOME ISN'T A PLACE, BUT THE PEOPLE THAT LIVE THERE. I THINK THE SAME APPLIES FOR MY EXPERIENCE AT NMSS.

While I enjoyed living on campus and was amazed by different fields and aspects of maths that does not get covered at school, it was the people that I met and the wonderful community bonded by a love for mathematics that I experienced, that I think will stick with me forever.

The maths that we did at camp was inherently different from anything that I had experienced before, it was messy, took a lot of experimentations, and you were not always guaranteed a neat solution (or a solution at all!). I valued this experience, as it really provided insight into what mathematics is like. It is, after all, not about what the solution is, but the exploration, discovery and learning process. We covered number theory and algorithms during the camp. Number theory takes you back to the basics of numbers, one example was when we constructed the set of integers from only axioms, it was this sort of focus on fundamentals that made me understand maths more.

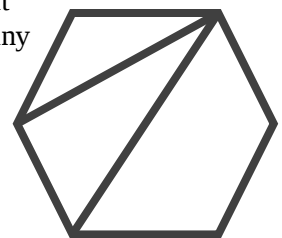
But it really was the people and community there that really made an impact on me. I had the valuable opportunity to meet so many like-minded people. It was always fun talking to people my age about the future, our experiences and about problems in question

sets. Everyone always has unique perspectives and approaches to problems that often inspired me in some way. I'm also very grateful for all the tutors and lectures and speakers, talking to them made me realise all the possible paths I could take with mathematics in the future.

“ But it was really the people and community there that really made an impact on me.

From the daily jokes during announcements, tutorials, “assassins”, and board games, NMSS was such a supportive and inclusive community that made me feel at home from the first night. I really loved this community that is brought together by our love for maths, everyone was eager to engage in more intellectual conversations, or just messing around. I feel like it was the perfect balance of learning and fun. NMSS gave me a valuable opportunity to be able to experience what being in a mathematical community feels like. And I am simply in love with it and had an unbelievable week.

Overall, NMSS was such a great experience that truly showed me what being a mathematician is like. It introduced me to so many interesting aspects of mathematics, allowed me to form so many valuable friendships and was overall an unforgettable experience.



Raymond Zhao, NMSS 2020 Student

A MARBLE QUANDARY

You have 12 marbles, completely identical in size, colour, and shape, but one of the marbles weighs differently (slightly more or slightly less, you don't know which). How can you determine which is the odd one out, using a set of balance scales that you can use only 3 times?



IS SCHOOL MATHS REALLY MATHS?

A first-year PhD student in Mathematics at Monash University, Emily has tutored at NMSS twice, in 2017 and 2019, and was a welfare officer in 2020. Emily also has a background in Education with a partial degree in the subject.

I loved maths at school. I was good at it. Which is to say, I was good at replicating the methods and strategies my teachers taught me, and more often than not, the answers I found matched the ones in the back of the textbook. I'm really glad that I enjoyed this kind of maths because it's the only reason I continued studying it at university. And I'm really glad that I kept enjoying maths when it became less about calculating and more about reasoning. My understanding of what maths is has changed a lot over the years.

“ *[Doing maths] is taking what we already know to be true and building on it, using logic and creativity* ”

Nowadays, I like to think of how we learn maths at school as comparable to how we learn languages. In primary school we learn fractions and our times tables. In high school, we're introduced to algebra and trigonometry. These foundations are important and necessary – they are the vocabulary and grammar of maths. It takes time and practice to become fluent; and if you don't master the basics, it's very easy to get left behind. Sadly, in both maths and in language learning, the most fun and creative parts only happen after we reach a certain level of fluency, and for many people this is far too late.

I won't go so far as to say that school maths is not maths. Instead, let me make the distinction between learning maths and doing maths. To my mind, learning maths is what happens when something that has been known for some time is taught to us as truth. Doing maths, on the other hand, is what happens when we don't know whether or not something is true. It is taking what we already know to be true and building on it, using logic and creativity – something most people would not associate with maths at all.

In my opinion, just because a person enjoys learning maths doesn't necessarily mean that they will enjoy doing maths. Just because a person picks up the vocabulary and grammar of a new language quickly or easily does not mean that they will suddenly want to write poetry in that language. I think there are parallels in maths. But I also believe that just because someone doesn't enjoy learning maths doesn't mean that they wouldn't enjoy doing maths. I wish that more people got to discover the joy of doing maths before they decided that 'maths is not for them'.

School maths looks very different to the maths done by mathematicians, which makes sense, because in school we are not yet fluent in the language. But it is certainly possible to replicate the experience of doing maths like a mathematician, even without a full mathematical toolkit. All we need is a bit of curiosity and a desire to notice patterns. It is my hope that we can create more space in the school curriculum for the doing of maths, as well as the learning of it. Maybe this would help more people to see maths in the way mathematicians see it: as a playful, exploratory, rewarding endeavour.

Emily Thompson, NMSS Welfare Officer

CREATIVE WRITING

A STUDENT COMPOSITION INSPIRED BY HIS EXPERIENCE AT NMSS 2020

ON THINKING DEEPLY OF SIMPLE THINGS

By Andrew Lim

Beauty in thought ... away from reality.
Far from the middling pastures of life ... and off to the unchartered seas,
To think deeply ... not to observe shallowly,
Not about manufactured constructs ... on simple things,
With greatest nemeses ... with fondest friends.

Here lies the grain and there lies the shore.
Only ever a few see the duty to explore:
It is we who then seek, thro' the ages, for truth.
Let that then be our call – for truth, truth eternal.
Our nemesis our love – the truth that eludes and slips away;
Our love our nemesis – the truth that reveals a whole new way.

tempus fugit ... *sed amor manet*
For the former ebbs and flows, transient. ... But the latter stands and holds, forever.
And it is always there – in the background – pushing on:
As Galois sits on his desk by night, mind the bullet, As Mirzakhani sits at hers by day,
writing furiously, running out of time. doodling her 'slow' work, chased by fate.

We run, we amble, perhaps it matters not –
Where'er we are midst paper forgot.
With friends and comrades by our side;
Taking NeMeSeS of years gone past in stride.

Go slow, go fast ... Chase one, chase many:
and the way – it shall appear. ... But “think deeply of simple things”
... and that way – it shall be clear.